Hydraulics of Nappe Flow Regime Above Stepped Chutes and Spillways

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SUMMARY Stepped chutes have become a popular method for discharging flood waters. The steps increase significantly the rate of energy dissipation taking place on the spillway face and reduce the size of the required downstream dissipation basin. In this paper, the author reviews the characteristics of nappe flows on stepped chutes. An analogy with drop structures enables the estimation of all the flow parameters along the chute. Calculations of the rate of energy dissipation are developed. The limiting conditions for nappe flow are also detailed.

The results indicates that nappe flow regime occurs for small discharges. Also the rate of energy dissipation along stepped channels with nappe flow regime increases with decreasing discharges.

NOMENCLATURE

The following symbols are used in this article:

- d flow depth (m);
- d_b flow depth at the brink of a step (m);
- d_c critical flow depth (m): for a rectangular channel: $d_c = \sqrt[3]{q_w^2/g}$
- d_i nappe thickness (m) at the impact with the receiving pool;
- d_p flow depth in the pool beneath the nappe (m);
- Fr Froude number defined as: $Fr = \dfrac{q_w}{\sqrt{g * d^3}}$;
- g gravity constant (m/s²);
- H total head (m);
- H_{dam} dam height (m);
- H_{max} maximum head available (m);
- H_o reservoir free-surface elevation (m) above the spillway crest;
- h step height (m);
- L_d length of the drop (m) measured from the vertical face of a step;
- L_r length (m) of the hydraulic jump roller;
- l horizontal length of steps (m);
- P_{atm} atmospheric pressure (Pa);
- Q discharge (m³/s);
- Q_{air}^{nappe} nappe ventilation (m³/s);
- q discharge per unit width (m²/s);
- t time (s);
- V velocity (m/s);
- V_c critical flow velocity (m/s);
- V_i nappe velocity (m/s) at the impact with the receiving pool;
- α spillway slope;
- ΔH head loss (m);
- ΔP nappe subpressure (Pa);
- Γ_y vertical acceleration (m/s²) of the flow at the brink of the step;
- θ angle of the falling nappe with the horizontal;
- ρ density (kg/m³);

Subscript

- w water flow;
- 1 at section 1 (fig. 2);
- 2 at section 2 (fig. 2).

1 INTRODUCTION

1.1 Presentation

Energy dissipation over dam spillways is usually achieved by a standard stilling basin at the downstream end of the spillway where a hydraulic jump is created to dissipate a large amount of energy, a high velocity water jet taking off from a flip bucket and impinging into a downstream plunge pool, or the construction of a stepped spillway to assist in energy dissipation. Water flowing over a stepped chute can dissipate a major proportion of its energy. The steps increase significantly the rate of energy dissipation taking place along the spillway face, and eliminate or reduce greatly the need for a large energy dissipator at the toe of the spillway.

Recently the development of new construction materials (e.g. roller compacted concrete RCC, gabions) has increased the interest for stepped spillways. The construction of stepped spillway is compatible with the slipforming and RCC placing methods. Also gabion stepped spillways are the most common type of spillways used for gabion dams.

Stepped cascades are utilised also in water treatment plants. As an example, five artificial cascades were designed along a waterway system to help the re-oxygenation of the polluted waters (GASPAROTTO 1992). The waterfalls were landscaped as leisure parks and combined flow aeration and aesthetics. Aesthetical applications of stepped cascades can include stepped fountains in cities (e.g in Brisbane, Hong Kong, Taipei).

1.2 Flow regimes

A stepped chute consists of an open channel with a series of drops in the invert. The total fall is divided into a number of smaller falls. The flow over stepped spillways can be divided into two regimes: nappe flow and skimming flow.

In the skimming flow regime, the water flows down the stepped face as a coherent stream, skimming over the steps and cushioned by the recirculating fluid trapped between them. The external edges of the steps form a pseudo-bottom over which the flow passes. Beneath this, horizontal axis vortices develop and are maintained through the transmission of shear stress from the water flowing past the edge of the steps. Characteristics of this flow regime were detailed elsewhere (e.g. RAJARATNAM 1990, CHANSON 1993a).

In nappe flow regime, the water proceeds in a series of plunges from one step to another. The flow from each step hits the step below as a falling jet, with the energy dissipation occurring by jet breakup in air, jet mixing on the step, and with the formation of a fully developed or partial hydraulic jump on the step. PEYRAS et al. (1991, 1992) indicated two types of nappe flow: 1- nappe flow with fully developed hydraulic jump (or isolated nappe flow), for low discharge and small flow depth, and 2- nappe flow with partially developed hydraulic jump (also called partial nappe flow) (fig. 1).

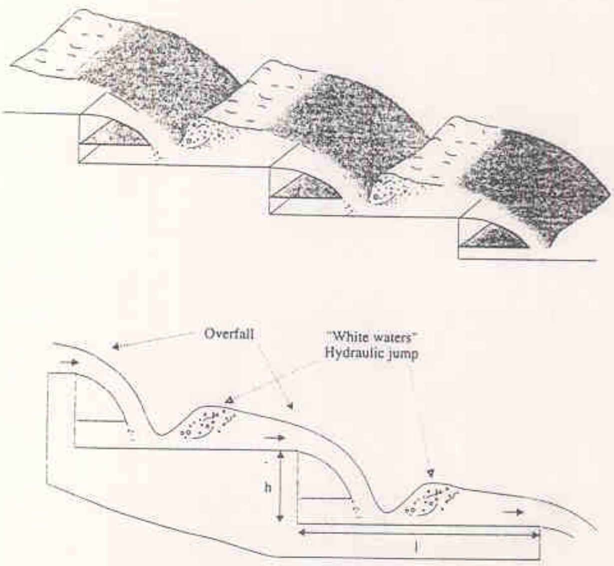

Figure 1 Nappe flow regime above a stepped spillway

Both the nappe flow regime and the skimming flow regime enable large rates of energy dissipation along a spillway (PEYRAS et al. 1991, CHANSON 1993a). In this paper, the author develops the flow characteristics of nappe flows. The calculations are compared with model data. Then, the energy dissipation performances are developed. Later, limiting flow conditions for nappe flow regime are presented.

It must be noted that this article presents results obtained for chutes with horizontal steps. ESSERY and HORNER (1978), PEYRAS et al. (1992) and FRIZELL (1992) discussed experimental results obtained with inclined and pooled steps. Further the author wishes to emphasise that nappe flow situations occur for shallow flows: i.e., small to medium floods. In the case of large flood discharges, it will be shown that a skimming flow regime takes place and the flow calculations are completely different (e.g. CHANSON 1993a).

1.3 Air entrainment in nappe flows

In spillway flows the amount of entrained air is an important design parameter. Air entrainment increases the bulk of the flow which is a design parameter that determines the height of spillway sidewalls. Also the presence of air within the boundary layer may reduce the shear stress between the flow layers and hence the flow resistance. Further the presence of air within high-velocity flows may prevent or reduce the damage caused by cavitation. Air entrainment on spillways and chutes has been recognised also for its contribution to the air-water transfer of atmospheric gases such as oxygen and nitrogen.

In a nappe flow regime, air bubble entrainment is observed at each step by the plunging jet at the intersection of the overfalling jet and the receiving waters and at the toe of the hydraulic jump (fig. 1). With deep pooled steps, most of the air is entrained by the plunging jet. Extensive studies of plunging jet entrainment were performed (e.g. VAN DE SANDE and SMITH 1973, ERVINE and ELSAWY 1975) and reviewed by WOOD (1991) and CHANSON and CUMMINGS (1992). For flat steps with shallow waters, most of the air is entrained at the toe of the hydraulic jumps. The air entrainment characteristics of hydraulic jumps were analysed by a number of researchers (e.g. RAJARATNAM 1967, RESCH et al. 1974). Note that, for deep pooled steps, air is entrained only at the intersection of the nappe with the receiving pool.

It is worth noting that, for large step heights (i.e. $h > 5$ m), air entrainment may occur along the upper and lower interfaces of the falling nappe. Experiments were performed by ERVINE and FALVEY (1987) on circular jets and CHANSON (1993b) with two-dimensional jets. These studies may provide useful information on the amount of air entrained along the interface. In most practical situations however, the effects of air entrainment along the nappe interfaces are small and can be neglected.

2 HYDRAULIC CHARACTERISTICS OF NAPPE FLOWS

Along a spillway, a nappe flow is characterised by a succession of free-fall jets impinging on the next step and followed by a fully developed or partially developed hydraulic jump (fig. 1). Stepped spillways with nappe flows can be analysed as a succession of drop structures.

MOORE (1943) and RAND (1955) analysed a single-step drop structure (fig. 2). For a horizontal step, the flow conditions near the end of the step change from subcritical to critical at some section a short distance back from the edge. The flow depth at the brink of the step d_b is:

$$d_b = 0.715 * d_c \qquad (1)$$

where d_c is the critical flow depth (ROUSE 1936). Downstream of the brink, the nappe trajectory can be computed using potential flow calculations, complex numerical methods or approximate methods as that developed by MONTES (1992).

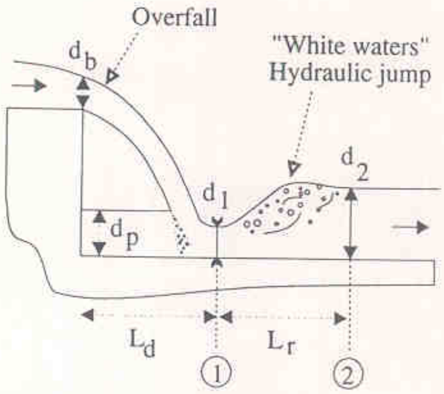

Figure 2 Flow at a drop structure

Application of the momentum equation to the base of the overfall leads to (WHITE 1943):

$$\frac{d_1}{d_c} = \frac{2^{1/2}}{\frac{3}{2^{3/2}} + \sqrt{\frac{3}{2} + \frac{h}{d_c}}} \qquad (2)$$

where d_1 is the flow depth at section 1 (fig. 2) and h is the step height. The total head H_1 at section 1 can be expressed non-dimensionally as:

$$\frac{H_1}{d_c} = \frac{d_1}{d_c} + \frac{1}{2} * \left(\frac{d_c}{d_1}\right)^2 \qquad (3)$$

The flow depth and total head at section 2 (fig. 2) are given by the classical hydraulic jump equations:

$$\frac{d_2}{d_1} = \frac{1}{2} * \left(\sqrt{1 + 8 * Fr_1^2} - 1\right) \qquad (4)$$

$$\frac{H_1 - H_2}{d_c} = \frac{(d_2 - d_1)^3}{4 * d_1 * d_2 * d_c} \qquad (5)$$

where Fr_1 is the Froude number defined at section 1:

$$Fr_1 = q_w / \sqrt{g * d_1^3}$$

RAND (1955) assembled several sets of experimental data and developed the following correlations:

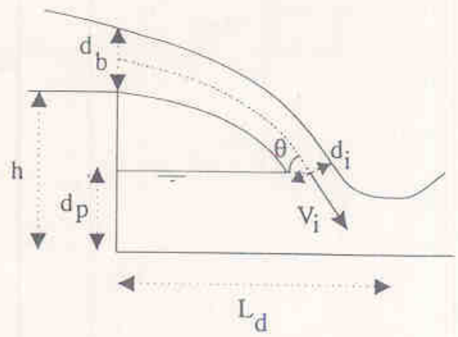

Figure 3 Nappe geometry

$$\frac{d_1}{h} = 0.54 * \left(\frac{d_c}{h}\right)^{1.275} \qquad (6)$$

$$\frac{d_2}{h} = 1.66 * \left(\frac{d_c}{h}\right)^{0.81} \qquad (7)$$

$$\frac{d_p}{h} = \left(\frac{d_c}{h}\right)^{0.66} \qquad (8)$$

$$\frac{L_d}{h} = 4.30 * \left(\frac{d_c}{h}\right)^{0.81} \qquad (9)$$

where d_p is the height of water in the pool behind the overfalling jet and L_d is the distance from the vertical face of the step to the position of the depth d_1 (fig. 2). Equation (6) is an empirical correlation that fits well equation (2). Note that equations (6) to (9) were obtained for an aerated nappe.

The flow conditions at the impact of the nappe with the receiving pool can be deduced from the equation of motion (appendix A). Using equations (1) and (8), the nappe thickness d_i, the nappe velocity V_i and the angle θ of the nappe with the horizontal, at the impact, can be correlated by:

$$\frac{d_i}{d_c} = 0.687 * \left(\frac{d_c}{h}\right)^{0.483} \qquad (10)$$

$$\frac{V_i}{V_c} = 1.455 * \left(\frac{d_c}{h}\right)^{-0.483} \qquad (11)$$

$$\tan\theta = 0.838 * \left(\frac{d_c}{h}\right)^{-0.586} \qquad (12)$$

Downstream of the impact of the nappe (fig. 2), the roller length of a fully-developed hydraulic jump is estimated as (HAGER et al. 1990):

$$\frac{L_r}{d_1} = 8 * (Fr_1 - 1.5) \qquad (13)$$

where L_r is the length of roller and d_1 and Fr_1 are the depth of flow and the Froude number immediately upstream of the jump: that is, at section 1.

If the length of the drop L_d plus the length of the roller L_r is smaller than the length of a step l, a fully developed hydraulic jump can take place (fig. 2). Combining equations (9) and (13), a condition for nappe flow regime with fully developed hydraulic jump is deduced: a nappe flow regime with fully-developed hydraulic jump occur for discharges smaller than a critical value defined as:

$$\left(\frac{d_c}{h}\right)_{char} = 0.0916 * \left(\frac{h}{l}\right)^{-1.276} \tag{14}$$

where l is the step length. Nappe flow situations with fully-developed hydraulic jump occur for $d_c/h<(d_c/h)_{char}$. Note that the correlation (eq. (14)) was obtained for: $0.2 \le h/l \le 6$.

Along a stepped spillway, critical flow conditions occur near to the end of each step, and equations (2) to (13) provide the main flow parameters for a nappe flow regime with fully developed hydraulic jump. PEYRAS et al. (1991, 1992) indicated that these equations can be applied also with reasonable accuracy to nappe flows with partially developed jumps.

3 ENERGY DISSIPATION

In a nappe flow situation with a fully developed hydraulic jump, the head loss at any intermediary step equals the step height. The total head loss along the spillway ΔH equals the difference between the maximum head available H_{max} and the residual head at the bottom of the spillway H_1 (eq. (3)). In dimensionless form, it yields:

$$\frac{\Delta H}{H_{max}} = 1 - \left(\frac{\frac{d_1}{d_c} + \frac{1}{2} * \left(\frac{d_c}{d_1}\right)^2}{\frac{3}{2} + \frac{H_{dam}}{d_c}}\right) \quad \text{Un-gated spillway (15a)}$$

$$\frac{\Delta H}{H_{max}} = 1 - \left(\frac{\frac{d_1}{d_c} + \frac{1}{2} * \left(\frac{d_c}{d_1}\right)^2}{\frac{H_{dam} + H_o}{d_c}}\right) \quad \text{Gated spillway (15b)}$$

where H_{dam} is the dam height and H_o is the reservoir free-surface elevation above the spillway crest. For an un-gated spillway, the maximum head available and the dam height are related by: $H_{max} = H_{dam} + 1.5^* d_c$. For a gated spillway: $H_{max} = H_{dam} + H_o$. The residual energy is dissipated at the toe of the spillway by a hydraulic jump in the dissipation basin. Combining equations (6) and (15a, 15b), the total energy loss becomes:

$$\frac{\Delta H}{H_{max}} = 1 - \left(\frac{0.54 * \left(\frac{d_c}{h}\right)^{0.275} + \frac{3.43}{2} * \left(\frac{d_c}{h}\right)^{-0.55}}{\frac{3}{2} + \frac{H_{dam}}{d_c}}\right) \quad \text{Un-gated spillway (16a)}$$

$$\frac{\Delta H}{H_{max}} = 1 - \left(\frac{0.54 * \left(\frac{d_c}{h}\right)^{0.275} + \frac{3.43}{2} * \left(\frac{d_c}{h}\right)^{-0.55}}{\frac{H_{dam} + H_o}{d_c}}\right) \quad \text{Gated spillway (16b)}$$

On figure 4, the head loss for an un-gated structure (eq. 16a) is plotted as a function of the critical flow depth and the number of steps, and compared with experimental data (MOORE 1943, RAND 1955, STEPHENSON 1979). Figure 4 indicates that most of the flow energy is dissipated on the stepped spillway for large dams (i.e. large number of steps). Note that, for a given dam height, the rate of energy dissipation decreases when the discharge increases. Figure 4 shows a good agreement between equation (16a) and model data obtained on a single step structure.

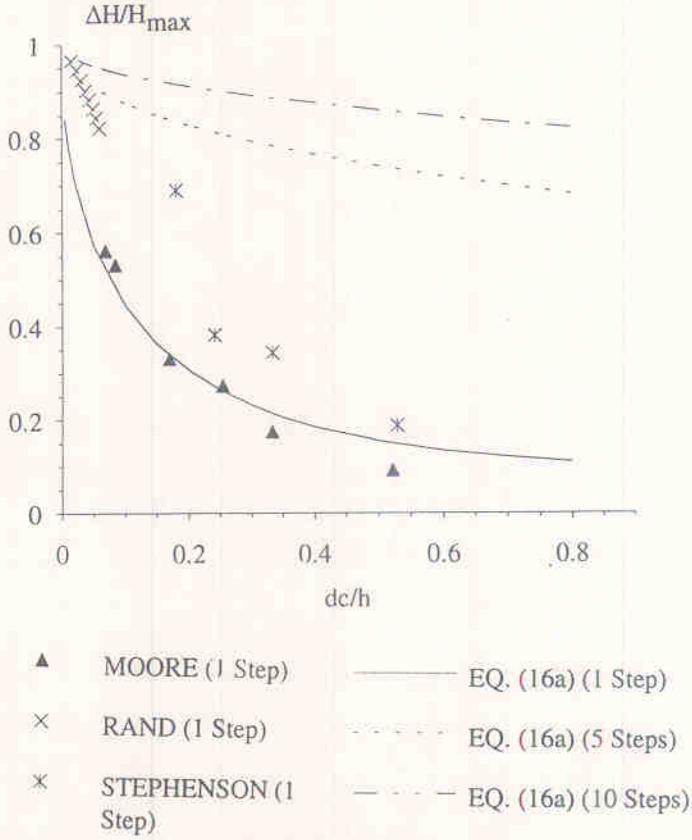

Figure 4 Energy dissipation in nappe flow regime-Comparison between equation (16) and experimental data (MOORE 1943, RAND 1955, STEPHENSON 1979)

Equations (15) and (16) were obtained for nappe flows with fully developed hydraulic jumps. PEYRAS et al. (1991) performed experiments for nappe flows with fully and partially developed hydraulic jumps. The rate of energy dissipation of nappe flows with partially developed hydraulic jumps was within 10% of the values obtained for nappe flows with fully developed hydraulic jump for similar flow conditions. Therefore, it is believed that equation (16) may be applied to most of the nappe flow situations with a reasonable accuracy.

4 AERATION OF THE NAPPE

When the falling nappe intersects the receiving pool of water, air is entrained within the flow. If the cavity between the nappe and the vertical step is not ventilated, the pressure in the cavity falls below atmospheric, with subpressure and nappe oscillations occurring. LEVIN (1968, pp. 28-37) gave indications of appropriate levels of nappe ventilation. The re-analysis of his data indicates that the nappe ventilation is estimated as:

$$\frac{Q_{air}^{nappe}}{Q_w} = 0.19 * \left(\frac{h-d_p}{d_b}\right)^{0.95} \quad \text{for } 3<Fr<10 \quad (17a)$$

$$\frac{Q_{air}^{nappe}}{Q_w} = 0.21 * \left(\frac{h-d_p}{d_b}\right)^{1.03} \quad \text{for } 13<Fr<15 \quad (17b)$$

where Q_{air}^{nappe} is the nappe aeration and Fr is a Froude number defined in term of the nappe thickness. Note that equations (17a) and (17b) apply to both bottom outlet aeration and nappe ventilation. For the ventilation of free-falling nappes LEVIN (1968) recommends the use of equation (17a). Figure 5 shows a comparison between equations (17) and LEVIN's (1968) data.

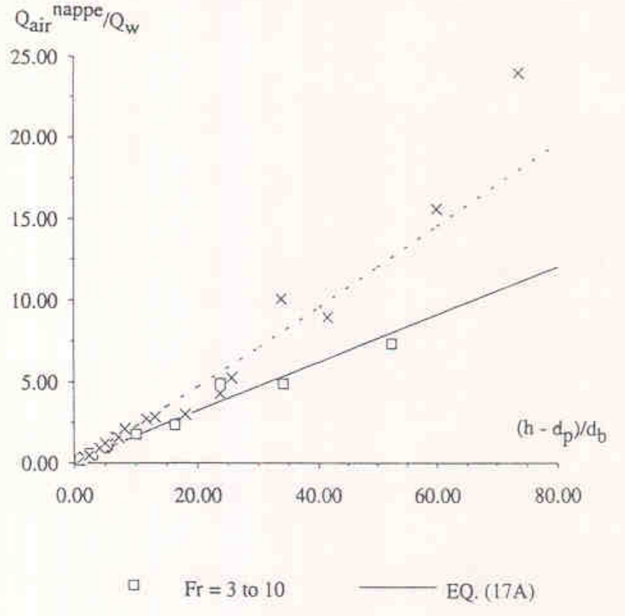

Figure 5 Nappe ventilation (LEVIN 1968)

5 LIMITING FLOW CONDITIONS OF NAPPE FLOW REGIME

5.1 Transition between nappe flow and skimming flow

For small discharges and flat slopes, the water flows as a succession of waterfalls (i.e. nappe flow regime). An increase of discharge or of slope might induce the nappe flow to skimming flow is a function of the discharge, the step height and length. The author re-analysed experimental data obtained by ESSERY and HORNER (1978), DEGOUTTE et al. (1992) and BEITZ and LAWLESS (1992). For these data, skimming flow regime occurs for discharges larger than a critical value defined as:

$$\frac{(d_c)_{onset}}{h} = 1.057 - 0.465 * \frac{h}{l} \quad (18)$$

where $(d_c)_{onset}$ is the characteristic critical depth. Figure 6 compares equation (18) with the experimental data. Nappe flow regime takes place for $d_c/h<(d_c)_{onset}$. Note that equation (14) is also shown on figure 6.

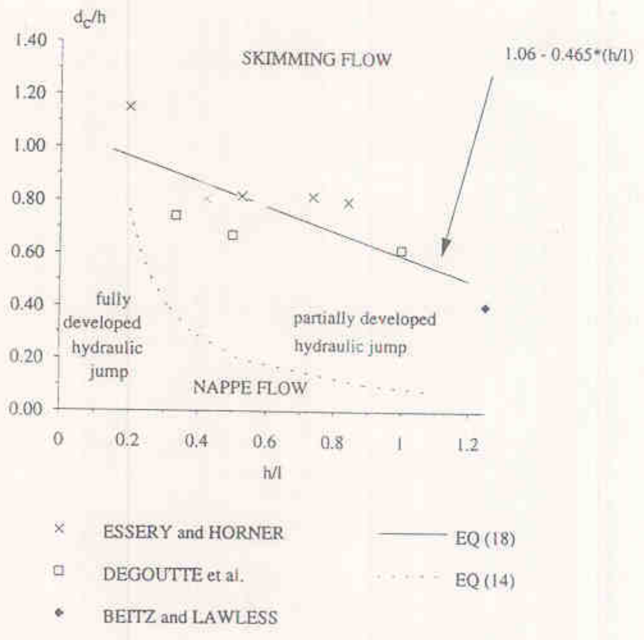

Figure 6 Transition from nappe flow to skimming flow regime

It must be noted that equation (18) was deduced for h/l ranging from 0.2 to 1.3. Further experimental work is required to verify (or to modify) equation (18) outside that range.

5.2 Design of chutes with nappe flow regime

A number of dams have been built in South Africa with stepped spillways. From this experience, STEPHENSON (1991) suggested that the most suitable conditions for nappe flow situations are:

$$\tan\alpha = \frac{h}{l} < 0.20 \quad (19)$$

and $\frac{d_c}{\ } \ \ 1$ (20)

Equation (19) and (20) satisfy equation (14). Hence the flow conditions defined by equations (19) and (20) correspond to a nappe flow regime with fully developed hydraulic jump.

For nappe flow regime, the steps need to be relatively large. This situation is not often practical but may apply to relatively flat spillways, streams and stepped channels. For steep spillways or small step heights, a skimming flow regime is more desirable and may achieve similar rate of energy dissipation.

6 EXAMPLE OF APPLICATION

Considering a stepped chute (10 steps) with nappe flow regime, the discharge per unit width is 1.1 m²/s. The dam height is 20 metres. The step height and length are: $h = 2$ m and $l = 7.46$ m. Equations (1) to (17) enable the computation of the hydraulic characteristics of the flow. The main results are given in table I.

Table I Application for $h = 2$ m, $q_w = 1.1$ m²/s, $H_{dam} = 20$ m (10 steps)

Variable (1)	Value (2)	Unit (3)	Eq. (4)	Comments (5)
d_c/h	0.25			Dimensionless critical depth
h/l	0.27			15- degree slope
$(d_c/h)_{char}$	0.49		(14)	Fully developed hydraulic jump
d_b	0.356	m	(1)	Flow depth at the brink of a step
d_i	0.175	m	(10)	Nappe thickness at the impact with the receiving pool
V_i	6.3	m/s	(11)	Impact velocity of the nappe
θ	62.2	degrees	(12)	Jet angle of the impinging nappe
d_1	0.183	m	(6)	Flow depth at start of hydraulic jump
Fr_1	4.5			Froude number at start of hydraulic jump
d_2	1.08	m	(7)	Flow depth downstream of hydraulic jump
d_p	0.80	m	(8)	Flow depth in pool beneath the nappe
L_d	2.8	m	(9)	Length of drop
L_r	4.36	m	(13)	Roller length of the hydraulic jump
$\Delta H/H_{max}$	90%		(16)	Rate of energy dissipation
Q_{air}^{nappe}/Q_w	0.60		(17)	Nappe ventilation

In summary: the flow is a nappe flow regime with fully-developed hydraulic jump. The rate of energy dissipation along the spillway is 0.90. The knowledge of the flow depth at any point along the chute enables an accurate design of the chute sidewalls.

7 CONCLUSION

On a stepped chute, two types of flow regime may occur: nappe flow and skimming flow. Limiting conditions of nappe flow situations are developed (eq. (18)) which indicate that nappe flow regime occurs for flat slopes and small discharges.

A nappe flow regime can occur with a fully developed or a partially developed hydraulic jump on each step. For nappe flows with fully-developed hydraulic jumps, the hydraulic characteristics and the energy dissipation performances are detailed. These results (eq. (1) to (17)) enable the calculation of all the flow characteristics at any position along the spillway. Energy dissipation calculations indicate that the rate of dissipation increases with the dam height. For a given dam height, however, the rate of energy dissipation decreases with increasing discharges.

8 ACKNOWLEDGEMENTS

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APPENDIX A - NAPPE FLOW TRAJECTORY

Nappe trajectory of aerated nappe

For a horizontal step, a simple expression of the nappe trajectory can be deduced from the motion equation assuming that the velocity of the flow at the brink of the step is horizontal. When the fluid leaves the step (fig. 3), the acceleration is vertical only and equals minus the gravity constant assuming an aerated nappe. The time t, taken to reach the pool free-surface is given by:

$$d_p = -\frac{1}{2} * g * t^2 + \left(h + \frac{d_b}{2}\right) \qquad \text{(A-1)}$$

where d_p is the height of water in the pool behind the overfalling jet, h is the step height, d_b is the flow depth at the brink of the step.

The nappe thickness d_i and the flow velocity V_i at the intersection of the falling nappe with the receiving pool are then:

$$\frac{d_i}{d_c} = \left(\left(\frac{d_c}{d_b}\right)^2 + 2 * \frac{h + \frac{d_b}{2} - d_p}{d_c}\right)^{-1/2} \qquad \text{(A-2)}$$

$$\frac{V_i}{V_c} = \sqrt{\left(\frac{d_c}{d_b}\right)^2 + 2 * \frac{h + \frac{d_b}{2} - d_p}{d_c}} \qquad \text{(A-3)}$$

The angle of the falling nappe with the horizontal (fig. 3) is given by:

$$\tan\theta = \sqrt{2} * \sqrt{\frac{d_b}{d_c}} * \sqrt{\frac{h + \frac{d_b}{2} - d_p}{d_c}} \qquad \text{(A-4)}$$

Using the results of ROUSE (1936) and RAND (1955) for d_b and d_p, equations (A-2) to (A-4) can be correlated as:

$$\frac{d_i}{d_c} = 0.687 * \left(\frac{d_c}{h}\right)^{0.483} \qquad \text{(A-5)}$$

$$\frac{V_i}{V_c} = 1.455 * \left(\frac{d_c}{h}\right)^{-0.483} \qquad \text{(A-6)}$$

$$\tan\theta = 0.838 * \left(\frac{d_c}{h}\right)^{-0.586} \qquad \text{(A-7)}$$

Nappe trajectory of un-aerated nappe

Considering an un-aerated nappe, the pressure in the cavity between the nappe and the step is $P_{atm} - \Delta P$, where P_{atm} is the atmospheric pressure. At the brink of the step, the vertical acceleration of the flow Γ_y equals:

$$\Gamma_y = -g * \left(1 + \frac{\Delta P}{\rho_w * g * d_b}\right) \qquad \text{(A-8)}$$

The complete jet trajectory can be deduced from the motion equation (e.g. SCHWARTZ and NUTT 1963, CHANSON 1988).

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